Analytical Calculation of Magnetic Field Created by a Ring Magnet Used in Magnetron RF Reactor

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***Abstract* – In this work, we present an analytical method for calculating the axial and radial components of the magnetic field created by a permanent magnet in the form of ring used for magnetron Radio Frequency (RF) reactor, the calculation is based on the complete elliptic integral of first kind and also heuman lambda function by using Linux 64-bit. This method allows us to accurately assess the magnetic field, and also allows us to reduce the computational time.**

Keywords – magnetron RF reactor, complete elliptic integral, Heuman lambda Function

I. Introduction

The magnetrons Radio Frequency (RF) discharges are used in particular for the deposition of thin metal or insulating layers for nano-components used in the field of micro-electronics, for surface treatment, and also, for the Deep Reactive Ion Etching (DRIE Technology) of the Silicon to manufacture capacitors with high density and the depositing thick insulation or thick metal to manufacture micro-coils. The references [1-6] are examples of theoretical and experimental work in this area. In these reactors, a permanent magnet is placed next to the cathode surface in order to make the electrons stay longer in the plasma, and also to confine these electrons before they are collected or recombined at the electrodes. This phenomenon allows increasing the plasma density [2] which is much lower in the absence of magnetic field due to the very low gas densities [3]. Under these conditions, the ions generated in the plasma effectively contribute to the cathodic sputtering of target and forming in the gas phase the necessary precursors to deposit the thin films on the substrate placed on the anode electrode. The optimization of such a deposit can be made using experimental diagnostic tools supplemented with tools for modeling the dynamics of the discharge and the kinetics of the non-equilibrium plasma formed. These models require absolutely a priori knowledge of the magnetic field that exists within the plasma reactor. That is why the purpose of this paper is to propose an analytical model for calculating the magnetic field created by one a permanent magnet ring radially magnetized to be used in the future in a particle model for the simulation of a magnetron reactor. Several analytical and numerical approaches have been made to the calculation of the magnetic field created by the magnets [7-11].

The main objective of this work is to calculate the axial and radial components of the magnetic field in the volume of the discharge created by a permanent magnet in a ring shape radially magnetized. This calculation is based on the complete elliptic integral of first kind also heuman lambda function. Figure 1 shows a scheme of a magnetron RF reactor, the domain of the calculation is defined by the volume of the reactor constituted by two parallel and circular electrodes of radius R_r and the distance between them is H_r, the distance between the cathode and the permanent magnet is $(H-h)$ where h is the amplitude of the magnet (see figure.2).

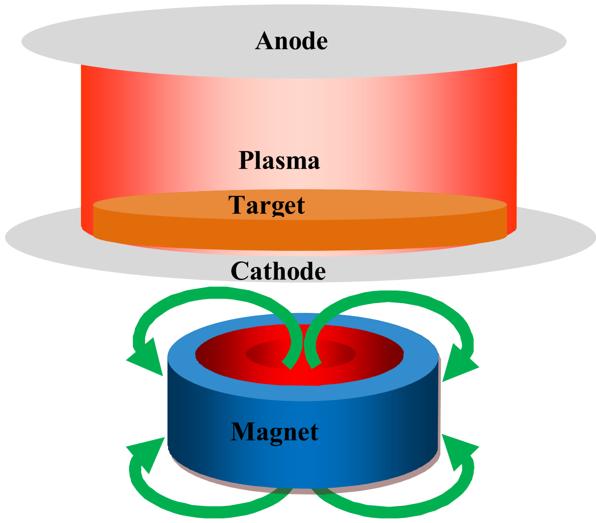

Fig. 1 Scheme of a magnetron RF reactor

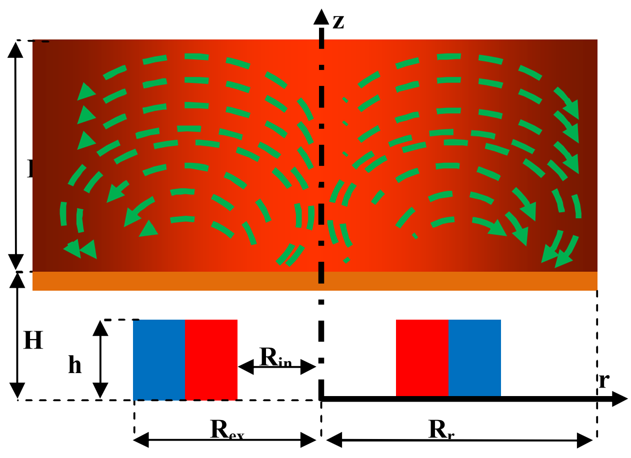

Fig.2. Scheme of design of the magnet with the reactor

II. BASIC EXPRESSIONS OF COMPUTING THE MAGNETIC FIELD

The calculation of the magnetic field created by a permanent magnet can be obtained by using the Coulomb method current model [8], [11]. In this paper we used the coulomb method, in the case of a radial magnetization (see fig.2), the magnetic field at any point of coordinates $M(r,z)$ created by the inner face of magnetization $+\sigma$ and outer face of magnetization $-\sigma$ respectively is given by the following expressions:

$$\vec{B}^{+}(r,z) = \frac{+\sigma}{4\pi}\cdot\int_{z_1=0}^{z_1=h}\int_{\theta=0}^{\theta=2\pi}\frac{\overrightarrow{PM}}{\left|\overrightarrow{PM}\right|^3}.r_1.dz_1.d\theta \quad (1)$$

$$\vec{B}^{-}(r,z) = \frac{-\sigma}{4\pi}\cdot\int_{z_1=0}^{z_1=h}\int_{\theta=0}^{\theta=2\pi}\frac{\overrightarrow{PM}}{\left|\overrightarrow{PM}\right|^3}.r_1.dz_1.d\theta \quad (2)$$

$$\vec{B}(r,z) = \vec{B}^{+}(r,z) + \vec{B}^{-}(r,z) \quad (3)$$

$$\overrightarrow{PM} = (r - r_1\cos\theta).\vec{i}_r - r_1\sin\theta\,\vec{i}_\theta + (z - z_1)\,\vec{i}_k \quad (4)$$

Where $P(r_1, z_1, \theta)$ is point located on the inner or outer face of the permanent magnet defined by its radial position r_1 ($r_1=R_{in}$ for the inner face and $r_1=R_{ex}$ for the outer face), axial position z_1 and polar angle θ and the equation (1) can be re-written as follows:

$$\vec{B}^{+}(r,z) = \frac{+\sigma}{4\pi}\cdot\int_{z_1=0}^{z_1=h}\int_{\theta=0}^{\theta=2\pi}\times$$
$$\left[(r - R_{in}\cos\theta).\vec{i}_r - R_{in}\sin\theta\,\vec{i}_\theta + (z - z_1)\,\vec{i}_k\right]\times$$
$$\frac{R_{in}.dz_1.d\theta}{\left|r^2 + R_{in}^2 - 2rR_{in} + (z - z_1)^2\right|^{\frac{3}{2}}} \quad (5)$$

In this equation, the first term represents the radial component of the magnetic field created by the inner face of the permanent magnet B_r^+, the second term represents the tangential component B_θ^+ and third term represents the axial component B_z^+. Due to the symmetry, the tangential component is zero ($B_\theta^+ = 0$). The same steps may be followed in order to find the radial component (B_r^-) and the axial component (B_z^-) created by the magnetization of the outer face of the magnet ring. By substituting the variable $\theta = \pi\text{-}2\varphi$ in the equation (5) and after some mathematical calculation we obtain the following expressions:

$$\vec{B}_r^{+}(r,z) = \frac{+\sigma}{4\pi}\cdot\sum_{i=1}^{2}(-1)^{i-1}\sqrt{\frac{R_{in}}{r}}\left[2\frac{t_i k_i^+}{r+R_{in}}K\left(k_i^+\right)+\right.$$
$$\left.\pi\sqrt{\frac{R_{in}}{r}}.sign\left((r-R_{in}).t_i\right)\left[1-\Lambda_0\left(\varepsilon_i^+,k_i^+\right)\right]\right] \quad (6)$$
$$\vec{B}_r^{-}(r,z) = \frac{-\sigma}{4\pi}\cdot\sum_{i=1}^{2}(-1)^{i-1}\sqrt{\frac{R_{ex}}{r}}\left[2\frac{t_i k_i^-}{r+R_{ex}}K\left(k_i^-\right)+\right.$$
$$\left.\pi\sqrt{\frac{R_{ex}}{r}}.sign\left((r-R_{ex}).t_i\right)\left[1-\Lambda_0\left(\varepsilon_i^-,k_i^-\right)\right]\right]$$

$$\vec{B}_z^{+}(r,z) = \frac{+\sigma}{4\pi}\cdot\sum_{i=1}^{2}(-1)^{i-1}\sqrt{\frac{R_{in}}{r}}.k_i^+.K\left(k_i^+\right) \quad (7)$$
$$\vec{B}_z^{-}(r,z) = \frac{-\sigma}{4\pi}\cdot\sum_{i=1}^{2}(-1)^{i-1}\sqrt{\frac{R_{ex}}{r}}.k_i^-.K\left(k_i^-\right)$$

$$k_i^+ = \sqrt{\frac{4.r.R_{in}}{\left(r+R_{in}\right)^2+t_i^2}}\;;\quad k_i^- = \sqrt{\frac{4.r.R_{ex}}{\left(r+R_{ex}\right)^2+t_i^2}}$$

$$h_i^+ = \frac{4.r.R_{in}}{\left(r+R_{in}\right)^2}\;;\quad h_i^- = \frac{4.r.R_{ex}}{\left(r+R_{ex}\right)^2}\;;t_1 = z-h\;;\; t_1 = z$$

$$\varepsilon_i^+ = \arcsin\sqrt{\frac{1-h^+}{1-k_i^+}}\;;\quad \varepsilon_i^- = \arcsin\sqrt{\frac{1-h^-}{1-k_i^-}}$$

$K(k)$ is the complete elliptic integral of first kind and $\Lambda_0(\beta,k)$ is the heuman lambda function where:

$$K(k) = R_F\left(0, 1-k^2, 1\right) \quad (8)$$

$$\Lambda_0(\beta,k) = \frac{2}{\pi}\frac{\left(1-k^2\right)\sin\beta\cos\beta}{\Delta}\times$$
$$\left[R_F\left(0,1-k^2,1\right)+\frac{k^2}{3\Delta^2}R_J\left(0,1-k^2,1,1-\frac{k^2}{\Delta^2}\right)\right] \quad (9)$$

R_F and R_J are respectively the symmetric integrals of first kind and third kind where:

$$R_F(x,y,z) = \frac{1}{2}\int_0^\infty\left[(t+x)(t+y)(t+z)\right]^{-\frac{1}{2}}dt$$
$$R_J(x,y,z,p) = \frac{3}{2}\int_0^\infty\left((t+x)(t+y)(t+z)\right)^{-\frac{1}{2}}(t+p)^{-1}dt \quad (10)$$

Where the square root is taken real and positive if x, y, z are positive and varies continuously when x, y, z become complex. The integral is well defined if x, y, z lie in the complex plane cut along the non positive real axis (henceforth called the “cut plane”), with the exception that at most one of x, y, z may be 0. The same requirements apply to the symmetric integral of the third kind, where $p \neq 0$ and the Cauchy principal value is to be taken if p is real and negative. A degenerate case of R_F that embraces the inverse circular and inverse hyperbolic functions.

The reader can find the algorithm of these kinds of integrals in the reference (12). For accurate results, especially at the vicinity of singular points, the code of calculation was performed under linux 64-bit.

III. SIMULATION RESULTS

The dimensions of the geometry used in this calculation are as follows, $H = 3.5$ cm, $h = 3$ cm, $R_r = 6$ cm, $H_r = 2.5$ cm, $R_{in} = 2$ cm and $R_{ex} = 4$ cm. The magnetization is considered constant $\sigma = 1$T for the inner face and $\sigma = -1$T for the outer face (1T = 10^4 Gauss).

In figure 3 and figure 4 respectively, we show the contours of radial component and axial component of magnetic field created by the permanent magnet ring in all volume shown in figure 2 and figure 5 shows the contours of the total magnetic field in the same volume. It is clear that the points which are located in the corners of the permanent magnet ring are singular points. These points are treated by using the Hospital rule. Figure 6 illustrate the spatial variation of the radial component of the magnetic field in the volume of

reactor and created by the permanent magnet ring radially magnetized, this component is calculated only in the bulk of reactor which located by (*H-h*) = 0.5 cm from the magnet ring in the upper side. This component is large in the vicinity of the magnetic ring and the location of the target. It is necessary to say that this component has the major effect on plasma confinement by making the particles charged stay longer in this region and as result of that, it is effectively contribute to the cathodic sputtering. Figure 7 shows the spatial variation of the axial component of the magnetic field in the volume of reactor and created by the permanent magnet ring radially magnetized, this component does not have an effect on the dynamic of plasma.

Finally, figure 8 shows the spatial variation of the total magnetic field in the bulk of the asymmetric magnetron reactor, it represents the magnitude of the total magnetic field.

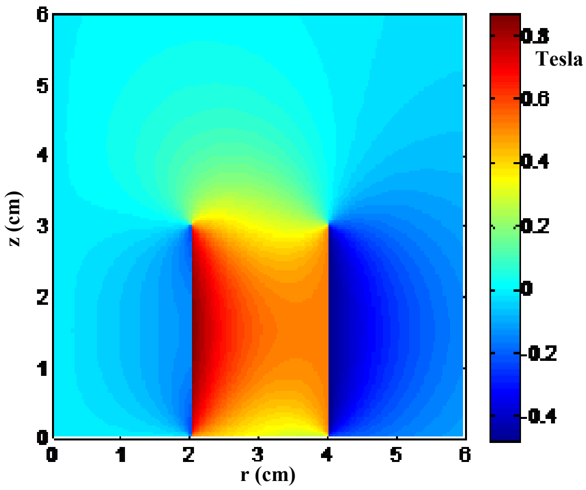

Fig.3. Radial component contours of magnetic field created by permanent magnet ring radially magnetized

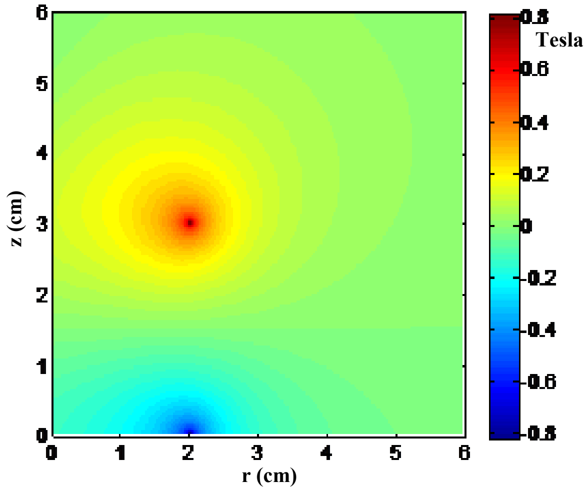

Fig.4. Axial component contours of magnetic field created by permanent magnet ring radially magnetized

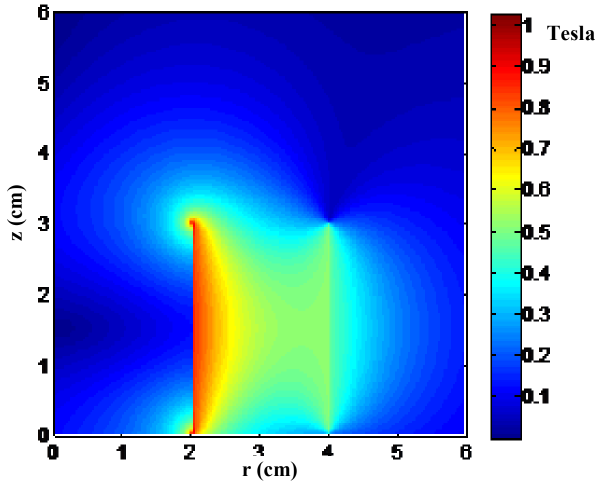

Fig.5. Contours of the total magnetic field created by permanent magnet ring radially magnetized

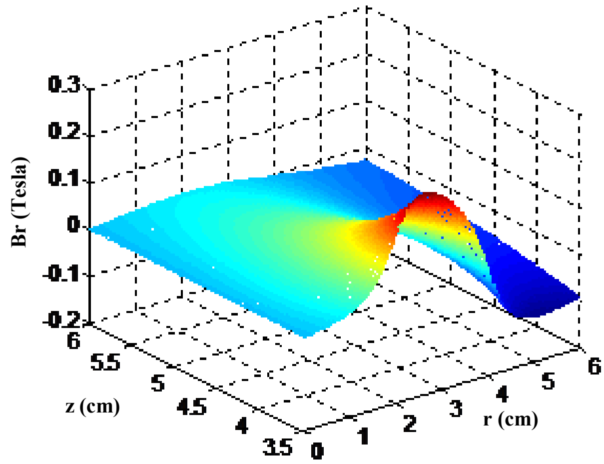

Fig.6. Spatial variation of the radial component of the magnetic field in the bulk of reactor

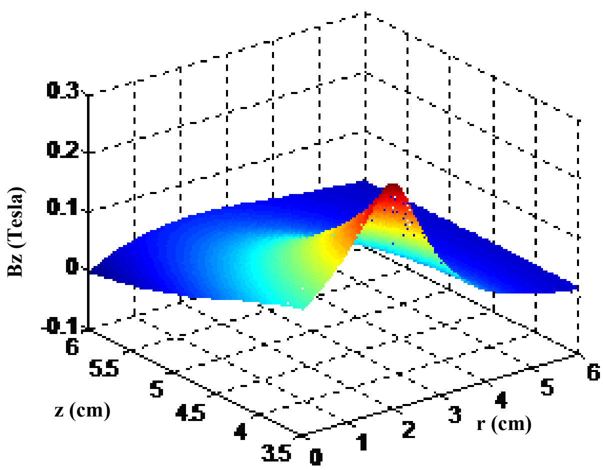

Fig.7. Spatial variation of the axial component of the magnetic field in the bulk of reactor

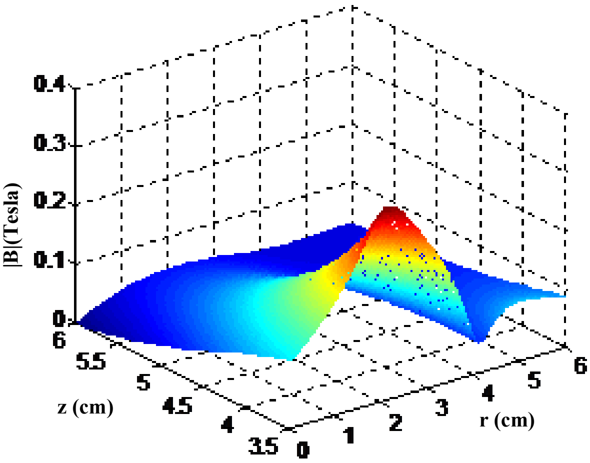

Fig.8. Spatial variation of the total magnetic field in the bulk of reactor

IV. Conclusion

In this paper we presented the analytical expressions for the calculation of the components of the magnetic field created by one permanent magnet ring, this magnet can be used in the magnetron reactor for the confinement of the plasma. These expressions are valid to calculate the magnetic field at any regular point in the space. This kind of calculation is simple but it is faster in comparison with the usual numerical calculations of magnetic field.

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